

Capacity Scaling Law by Multiuser Diversity in Cognitive Radio Systems

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Abstract—This paper analyzes the multiuser diversity gain in a cognitive radio (CR) system where secondary transmitters opportunistically utilize the spectrum licensed to primary users only when it is not occupied by the primary users. To protect the primary users from the interference caused by the missed detection of primary transmissions in the secondary network, minimum average throughput of the primary network is guaranteed by transmit power control at the secondary transmitters. The traffic dynamics of a primary network are also considered in our analysis. We derive the average achievable capacity of the secondary network and analyze its asymptotic behaviors to characterize the multiuser diversity gains in the CR system.

I. INTRODUCTION

The demand for wireless spectrum is constantly increasing as various wireless communication services have appeared. Correspondingly the available spectrum has become scarce. Current spectrum allocation policies aggravate spectrum scarcity since a particular spectrum is dedicated to only specific users as Federal Communications Commission (FCC) reported [1]. As a solution to improve the spectrum efficiency, *cognitive radio* (CR) has emerged, where secondary (unlicensed) users try to access the spectrum without interfering with communications of primary users. The term CR is classified into several techniques depending on the way to access the spectrum licensed to the primary users: *overlaid*, *underlaid*, and *interweaved* CR systems [2]. Our work focus on interweaved CR system where the secondary users are allowed to access the spectrum licensed to primary users only when primary users do not occupy the spectrum. This approach comes from the idea of *opportunistic communication* [3]. Secondary users monitor whether the spectrum is occupied by primary users in order to opportunistically communicate over vacant spectrum without interfering with primary users. Stable throughput of an interweaved CR system was analyzed by reflecting random packet arrival of a primary user using queueing process in [4] and [5]. Srinivasa and Jafar [6] studied the optimum number of secondary users that maximizes the total throughput in a decentralized CR system.

Secondary user scheduling and medium access control play key roles in CR systems but they have not been well studied yet. It has been well known that opportunistic user selection for transmission provides a multiuser diversity gain due to fluctuations of fading channels [9]. For non-CR systems, there have been many studies on characterizing the multiuser diversity gains [10]–[13]. These studies showed that the multiuser diversity gain in terms of average capacity grows like

$\log_2(\ln N)$ and $\sqrt{2 \log_2 N}$ in Rayleigh fading channels and lognormal shadowing channels, respectively, where N is the number of users. The multiuser diversity gain of a secondary network in an underlaid CR system was recently discussed in [14], [15]. It was shown that the average capacity of the secondary network scales like $\log_2(\ln N)$ and $\log_2 N$ under the finite and the infinite peak transmit power constraints at the secondary transmitters, respectively. Moreover, the scaling laws of underlaid CR system in a cognitive ad hoc network was studied in [16] although it did not focus on the multiuser diversity gain coming from opportunistic user selection. Despite the fact that the term CR typically refers to interweaved CR systems, however, the multiuser diversity gain in interweaved CR systems has not been identified.

In this paper, we investigate characteristics of multiuser diversity gains in interweaved CR systems by deriving average achievable capacity of a secondary network. Multiuser scheduling and transmit power control are employed in a secondary network to maximize the achievable capacity of a secondary user network and to satisfy a quality of service (QoS) constraint of a primary user network. Unlike underlaid CR systems, secondary users are not allowed to access the spectrum licensed to primary users when a secondary network detects primary users' transmission. So our analysis reflects both traffic dynamics of a primary user network and sensing reliability in a secondary network. Furthermore, this paper takes into account the interference from a primary transmitter contrary to [14]. Our asymptotic analysis and numerical results insightfully capture the key characteristics of multiuser diversity gains in interweaved CR systems. It is shown that multiuser diversity gains in interweaved CR systems are quite different from those in non-CR based systems due to the QoS constraint on a primary user network. Moreover, even if the secondary users of interweaved and underlaid CR systems have different ways to access the spectrum licensed to primary users, the multiuser diversity gains of both CR systems show very similar asymptotic characteristics.

II. SYSTEM DESCRIPTION

A. System and Channel Model

The system model considered in this paper is illustrated in Fig. 1, which consists of a primary receiver (primary base station), a primary transmitter, a secondary receiver (secondary base station), and N secondary transmitters which try to

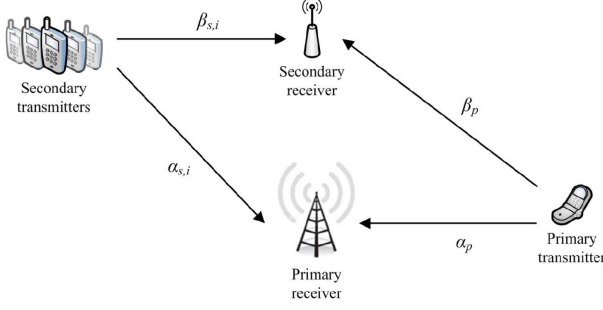


Fig. 1. System model

access the spectrum licensed to a primary transmitter. The additive thermal noise is assumed to be zero mean mutually independent, circularly symmetric, complex Gaussian random variable with unit variance. The channel gains from the i th secondary transmitter to the primary receiver and to the secondary receiver are denoted by $\alpha_{s,i}$ and $\beta_{s,i}$, respectively. Similarly, α_p and β_p are the channel gains from the primary transmitter to the primary receiver and to the secondary receiver, respectively. All channel gains are assumed to be independent and identically distributed (i.i.d.) exponential random variable with unit mean in Rayleigh flat fading channels. It is also assumed that the i th secondary transmitter knows the instantaneous channel state information of $\alpha_{s,i}$ and $\beta_{s,i}$ and knows the statistical channel information of α_p and β_p . The secondary transmitters are able to obtain the instantaneous channel state information of $\alpha_{s,i}$ and $\beta_{s,i}$ through a periodic sensing of pilot signal from the primary and secondary receivers, according to the channel reciprocity.

In our system model, all the secondary transmitters which have packets to transmit compute the maximum allowable transmit power $P_{s,i}$ based on $\alpha_{s,i}$ to maximize their throughput while ensuring the QoS of primary users. Then, the secondary transmitters report their 1-bit sensing results with calculated transmit power $P_{s,i}$. The secondary receiver fuses received sensing results from the secondary transmitters and makes a global decision. The reliability of cooperative sensing is captured by detection probability p_d and false alarm probability p_f . However, detailed discussion about how to fuse the received sensing results and how to reflect the performance of cooperative sensing are beyond the scope of this paper. If the global decision declares the absence of primary users' traffic in the spectrum, the secondary receiver selects the secondary transmitter which has the largest signal-to-noise power ratio (SNR) $\beta_{s,i}P_{s,i}$ to maximize secondary network throughput. Otherwise, all secondary transmitters are silent during a unit time slot. On the other hand, a primary transmitter transmits its packets with its maximum power, P_p .

B. Traffic Model

We consider a *packet-by-packet* access network where the spectrum licensed to the primary users is accessed in a time slotted manner similarly in [4]. In other words, all primary and secondary transmitters can transmit one packet

per each time slot. The traffic at the primary transmitter is modeled by a Bernoulli process with average packet arrival rate, $0 \leq \lambda \leq 1$ [packets/slot], and departure rate, $0 \leq \mu \leq 1$ [packets/slot]. Note that the departure rate can be interpreted as the probability of successful packet transmission during a time slot; moreover, it can directly be translated into average throughput of a primary user. In contrast to arrival rate λ , departure rate μ is significantly affected by the behavior of the secondary network. So the primary user sets a minimum tolerable departure rate μ_{\min} to guarantee a certain service rate for the case of missed detection in the secondary network by

$$\mu \geq \mu_{\min}. \quad (1)$$

It should be noted that the departure rate without any interference is greater than the arrival rate for an essentially stable primary user's queue.

III. QOS CONSTRAINT AND SECONDARY USER SCHEDULING

A. Departure Rate at a Primary Transmitter

The outage probabilities when there is no interference and when there is interference from the secondary network are given, respectively, by

$$P_{out} = \Pr [\log_2 (1 + \alpha_p P_p) < R] \quad (2)$$

$$P_{out}^{inf} = \Pr \left[\log_2 \left(1 + \frac{\alpha_p P_p}{1 + \alpha_{s,i} P_{s,i}} \right) < R \right], \quad (3)$$

where R is the required rate at the primary receiver. Then, the departure rate of the primary transmitter is given by

$$\mu = p_d(1 - P_{out}) + (1 - p_d)(1 - P_{out}^{inf}). \quad (4)$$

From a perspective of a primary user, a minimum departure rate, μ_{\min} , needs to be set as a OoS parameter and is notified to the secondary network so that interference from a secondary network is regulated to guarantee the minimum departure rate.

B. Power Adaptation and Secondary User Scheduling

Based on the channel information including the instantaneous value of $\alpha_{s,i}$ and the statistical properties of α_p , each secondary transmitter can estimate the corresponding outage probabilities of the primary user, when it is scheduled. The outage probabilities, (2) and (3), can be rewritten by

$$P_{out} = \Pr \left[\alpha_p < \frac{2^R - 1}{P_p} \right] = 1 - e^{-R_p} \quad (5)$$

$$P_{out}^{inf} = \Pr \left[\alpha_p < (2^R - 1) \frac{1 + \alpha_{s,i} P_{s,i}}{P_p} \right] \\ = 1 - e^{-R_p(1 + \alpha_{s,i} P_{s,i})} \quad (6)$$

where $R_p = (2^R - 1)/P_p$. If the i th secondary transmitter is scheduled for transmission, the departure rate of the primary user is obtained by plugging (5) and (6) into (4),

$$\mu(\alpha_{s,i}, P_{s,i}) = p_d e^{-R_p} + (1 - p_d) e^{-R_p(1 + \alpha_{s,i} P_{s,i})}. \quad (7)$$

The departure rate of the primary user is affected by transmit power of the secondary transmitter $P_{s,i}$ and instantaneous interference channel gain $\alpha_{s,i}$. Hereinafter, we use $\mu(\alpha_{s,i}, P_{s,i})$ instead of μ to represent the departure rate as a function of $\alpha_{s,i}$ and $P_{s,i}$.

The transmit power of the i th secondary transmitter is determined by the value making $\mu(\alpha_{s,i}, P_{s,i})$ equal to μ_{\min} since $\mu(\alpha_{s,i}, P_{s,i})$ is inversely proportional to $P_{s,i}$. The power $P_{\mu,i}$ corresponding to μ_{\min} is given by

$$P_{\mu,i} = \frac{1}{\alpha_{s,i}} \left[\frac{1}{R_p} \ln \left(\frac{1 - p_d}{\mu_{\min} - p_d e^{-R_p}} \right) - 1 \right]. \quad (8)$$

Since the transmit power of the secondary user is limited by a maximum power constraint, i.e., $P_{s,i} \leq P_{s,\max}$, the transmit power of the i th secondary transmitter is given by

$$P_{s,i} = \begin{cases} 0, & P_{\mu,i} < 0 \\ P_{\mu,i}, & 0 \leq P_{\mu,i} < P_{s,\max} \\ P_{s,\max}, & P_{\mu,i} \geq P_{s,\max} \end{cases}. \quad (9)$$

The first case $P_{\mu,i} < 0$ corresponds to the situation where the primary user cannot achieve μ_{\min} even if a secondary network do not interfere with the primary user. In this case, all the secondary transmitters can not transmit anything, regardless of the sensing result, so that there is no secondary network throughput. In the other cases, a secondary transmitter can transmit a packet with adaptively adjusted power based on the value of $\alpha_{s,i}$. Our analysis focuses on the last two cases where the spectrum licensed to primary user is opportunistically accessed. Hence, from (8), it is assumed that

$$K = \frac{1}{R_p} \ln \left(\frac{1 - p_d}{\mu_{\min} - p_d e^{-R_p}} \right) - 1 > 0 \quad (10)$$

where a positive real constant K is pre-determined by the system parameters.

Based on the computed $P_{s,i}$ in (9), all the secondary transmitters report their 1-bit sensing results with its transmit power, $P_{s,i}$. Using the collected sensing results, the secondary receiver checks the availability of the spectrum licensed to the primary network. If the spectrum is decided to be vacant, the secondary receiver selects the i^* th secondary transmitter whose SNR is the largest among N secondary transmitters such as

$$i^* = \arg \max_i \gamma_{s,i} \quad (11)$$

where the received SNR $\gamma_{s,i}$ from the i th secondary transmitter is given by

$$\gamma_{s,i} = \begin{cases} P_{\mu,i} \beta_{s,i} = \frac{K \beta_{s,i}}{\alpha_{s,i}}, & P_{\mu,i} < P_{s,\max} \\ P_{s,\max} \beta_{s,i}, & P_{\mu,i} \geq P_{s,\max} \end{cases}. \quad (12)$$

IV. CHARACTERISTICS OF MULTIUSER DIVERSITY IN A COGNITIVE RADIO SYSTEM

Unfortunately, analyzing the average capacity of secondary network is not easy to handle because extracting the PDFs of α_{s,i^*} and β_{s,i^*} from that of γ_{s,i^*} is intractable. Instead, to see

the exact scaling law of multiuser diversity gain, this section provides lower and upper bounds on the asymptotic average achievable capacity of the secondary network and derives the scaling law of the average achievable capacity of the secondary network from the two bounds.

A. A Lower Bound

To give the independency between $\alpha_{s,i}$ and $\beta_{s,i}$ in the selection of a secondary transmitter, scheduling procedure is divided into two stages. In the first stage, the secondary receiver determines a set \mathcal{S} consisting of the candidates for transmission which have low enough $\alpha_{s,i}$ so that the calculated $\max_i P_{\mu,i} > P_{s,\max}$, and hence the transmit power is saturated with $P_{s,\max}$. In the second stage, if the set \mathcal{S} is not empty, the i^\dagger th secondary transmitter which has the largest $\beta_{s,i}$ among the elements of \mathcal{S} is finally scheduled with its maximum allowable transmit power $P_{s,\max}$. Otherwise, the i^\dagger th secondary transmitter which has the maximum $\gamma_{s,i}$ among the N secondary transmitters is scheduled with the transmit power of P_{μ,i^\dagger} . Hence, the index of the scheduled secondary transmitter is given by

$$i^\dagger = \begin{cases} \arg \max_i \gamma_{s,i}, & \max_i P_{\mu,i} \leq P_{s,\max} \\ \arg \max_{i \in \mathcal{S}} \beta_{s,i}, & \max_i P_{\mu,i} > P_{s,\max} \end{cases}. \quad (13)$$

This two-stage scheduling certainly provides a lower bound on the average achievable capacity of the secondary network, which is given in the following theorem.

Theorem 1: For finite $P_{s,\max}$, a lower bound on the average achievable capacity of the secondary network converges to

$$\begin{aligned} \mathbb{E}[C_{s,low}] = & e^{\frac{K}{P_{s,\max} \ln(1-\frac{1}{N})}} \left[\frac{\lambda}{\mu_{\min}} (1 - p_d) \left(\log_2(1 + b_{N,low1}) \right. \right. \\ & \left. \left. - \frac{e^{\frac{1}{R_p}} E_1\left(\frac{1}{R_p}\right)}{\ln 2} \right) + \left(1 - \frac{\lambda}{\mu_{\min}} \right) (1 - p_f) \log_2(1 + b_{N,low1}) \right] \\ & + \left(1 - e^{\frac{K}{P_{s,\max} \ln(1-\frac{1}{N})}} \right) \left[\frac{\lambda}{M_{avg,l}} (1 - p_d) \right. \\ & \times \left(\log_2 \left(1 + P_{s,\max} \ln \left(N \left(1 - e^{-\frac{K}{P_{s,\max}}} \right) \right) \right) \right. \\ & \left. \left. - \frac{e^{\frac{1}{R_p}} E_1\left(\frac{1}{R_p}\right)}{\ln 2} \right) + \left(1 - \frac{\lambda}{M_{avg,l}} \right) (1 - p_f) \right. \\ & \left. \left. \times \log_2 \left(1 + P_{s,\max} \ln \left(N \left(1 - e^{-\frac{K}{P_{s,\max}}} \right) \right) \right) \right] \right] \quad (14) \end{aligned}$$

as N goes to infinity, where

$$b_{N,low1} = P_{s,\max} \mathcal{W} \left(\frac{KN}{P_{s,\max}} e^{\frac{K}{P_{s,\max}}} \right) - K, \quad (15)$$

$$M_{avg,l} = p_d e^{-R_p} + (1 - p_d) \frac{\left(e^{\frac{K(R_p P_{s,\max} + 1)}{P_{s,\max}}} - 1 \right) e^{-R_p(1+K)}}{\left(e^{\frac{K}{P_{s,\max}}} - 1 \right) (R_p P_{s,\max} + 1)} \quad (16)$$

where $\mathcal{W}(\cdot)$ denotes a Lambert W function.

Proof: The proof is given in Appendix. ■

As shown in (14), the lower bound can be divided into the capacities when the transmit power of the scheduled user is not bounded and when the transmit power is bounded by $P_{s,\max}$. The following corollary further simplifies the lower bound.

Corollary 1: If we assume that N goes to infinity, the result of Theorem 1 is approximated as

$$\begin{aligned}\mathbb{E}[C_{s,\text{low}}] &\approx \frac{\lambda}{M_{\text{avg},l}}(1-p_d)\log_2(\ln N) \\ &\quad + \left(1 - \frac{\lambda}{M_{\text{avg},l}}\right)(1-p_f)\log_2(\ln N) \\ &= \left[\frac{\lambda}{M_{\text{avg},l}}(1-p_d) + \left(1 - \frac{\lambda}{M_{\text{avg},l}}\right)(1-p_f)\right]\log_2(\ln N).\end{aligned}\quad (17)$$

Proof: In (14), the probability of a secondary transmission with unbounded power, $e^{\frac{K}{P_{s,\max}\ln(1-\frac{1}{N})}}$, decreases with N . ■

Corollary 1 indicates that the scaling law of the lower bound is $k_l \log_2(\ln N)$, where $0 \leq k_l \leq 1$ is constant determined by system parameters.

B. An Upper Bound

An upper bound can be obtained from the case that $\max_i \beta_{s,i}$ and $\min_j \alpha_{s,j}$ are the effective forwarding and interference channel gains of the scheduled secondary transmitter, respectively. This scenario certainly constructs an upper bound on the achievable capacity of the secondary network and the effective SNR of the scheduled secondary transmitter in this case is given by

$$\gamma_{s,up} = \begin{cases} \frac{K \max_i \beta_{s,i}}{\min_j \alpha_{s,j}}, & \max_i P_{\mu,i} \leq P_{s,\max} \\ P_{s,\max} \max_i \beta_{s,i}, & \max_i P_{\mu,i} > P_{s,\max} \end{cases}. \quad (18)$$

The following theorem shows an asymptotic behavior of the upper bound on the average achieved capacity.

Theorem 2: For finite $P_{s,\max}$, the upper bound of the secondary average achievable capacity converges to

$$\begin{aligned}\mathbb{E}[C_{s,up}] &= e^{\frac{K}{P_{s,\max}\ln(1-\frac{1}{N})}} \left[\frac{\lambda}{\mu_{\min}}(1-p_d) \left(\log_2(P_{s,\max} \ln N) \right. \right. \\ &\quad \left. \left. - \frac{E_1\left(\frac{1}{P_p}\right) e^{\frac{1}{P_p}}}{\ln 2} \right) + \left(1 - \frac{\lambda}{\mu_{\min}}\right)(1-p_f) \log_2(P_{s,\max} \ln N) \right] \\ &\quad + \left(1 - e^{\frac{K}{P_{s,\max}\ln(1-\frac{1}{N})}}\right) \left[\frac{\lambda}{M_{\text{avg},u}(N)}(1-p_d) \right. \\ &\quad \times \left(\log_2(1 + P_{s,\max} \ln N) - \frac{E_1\left(\frac{1}{P_p}\right) e^{\frac{1}{P_p}}}{\ln 2} \right) \\ &\quad \left. + \left(1 - \frac{\lambda}{M_{\text{avg},u}(N)}\right)(1-p_f) \log_2(1 + P_{s,\max} \ln N) \right] \quad (19)\end{aligned}$$

as N grows to infinity, where

$$\begin{aligned}M_{\text{avg},u}(N) &= p_d e^{-R_p} + (1-p_d) \frac{N e^{-R_p(1+K)} \left(e^{\frac{K(R_p P_{s,\max} + N)}{P_{s,\max}}} - 1 \right)}{(R_p P_{s,\max} + N) \left(e^{\frac{KN}{P_{s,\max}}} - 1 \right)}.\end{aligned}\quad (20)$$

Proof: The proof is similar to that of theorem 1. ■

The asymptotic upper bound on the average achievable capacity can be divided into the two parts according to the type of received SNR described in (18). The following corollary further simplifies the lower bound.

Corollary 2: If we assume that N goes to infinity, the asymptotic upper bound on the average achievable capacity in (19) is given by

$$\begin{aligned}\mathbb{E}[C_{s,up}] &\approx \frac{\lambda}{e^{-R_p}}(1-p_d)\log_2(\ln N) \\ &\quad + \left(1 - \frac{\lambda}{e^{-R_p}}\right)(1-p_f)\log_2(\ln N) \\ &= \left[\frac{\lambda}{e^{-R_p}}(1-p_d) + \left(1 - \frac{\lambda}{e^{-R_p}}\right)(1-p_f)\right]\log_2(\ln N).\end{aligned}\quad (21)$$

Proof: In (19), the capacity via bounded secondary transmit power, $\gamma_{s,up} = P_{s,\max} \max_i \beta_{s,i}$, becomes more dominant than the other as N grows. ■

Corollary 2 indicates that the upper bound grows like $k_u \log_2(\ln N)$, where $0 \leq k_u \leq 1$ is a constant determined by system parameters. Consequently, we can characterize the multiuser diversity gain of the secondary network under finite $P_{s,\max}$ from the scaling law of the lower and upper bounds. The asymptotic capacity of the secondary network is laid between the asymptotic lower and upper bounds as

$$k_l \log_2(\ln N) \leq \mathbb{E}[C_s] \leq k_u \log_2(\ln N). \quad (22)$$

This result implies that the scaling law of secondary network is $k \log_2(\ln N)$, where $k_l \leq k \leq k_u$. This scaling law is similar to that in underlaid CR system, $\log_2(\ln N)$, except a constant scaling term k determined by system parameters [14].

Fig. 2 shows the average achievable capacity of the secondary network, the lower bound, and the upper bound versus N for $P_{s,\max} = P_p$. It is verified that the lower and upper bounds on the secondary network capacity are quite well characterized by the asymptotic approximations even if the number of secondary transmitter is small. In addition, this figure confirms that the scaling laws of lower and upper bounds are the same as that of the exact capacity.

V. CONCLUSION

This paper has investigated the multiuser diversity gain and its capacity scaling law in an interweaved CR system. We have analyzed the capacity of a secondary network by taking into account the traffic dynamics of a primary user and reliability of

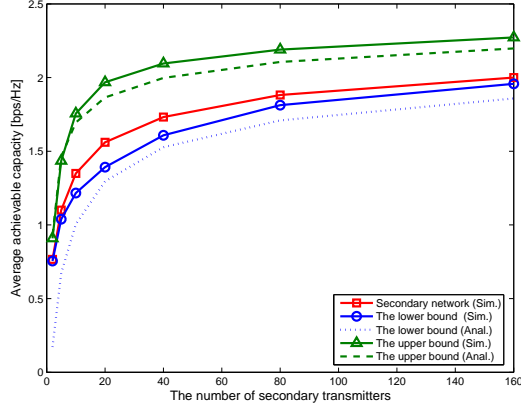


Fig. 2. Average capacities of the secondary network ($p_d = 0.8, p_f = 0.3, \lambda = 0.5, P_p = 10\text{dB}, P_{s,\max} = P_p, R = 0.5, \mu_{\min} = 0.95$)

spectrum sensing in the secondary network. Then, by applying extreme value theory, we have derived asymptotic capacity of the secondary network and characterized the multiuser diversity gain. The asymptotic capacity has been shown to grow like $k \log_2(\ln N)$, where $0 \leq k \leq 1$. This scaling law of the capacity is similarly observed in underlaid CR systems even though the mechanism of spectrum access is totally different. Our numerical results have also verified that the asymptotic capacity agrees well with the exact capacity even for small number of secondary transmitters, N .

APPENDIX PROOF OF THEOREM 1

The average achievable capacity by the suboptimal two stage scheduling is given by

$$\mathbb{E}[C_{s,\text{low}}] = \Pr \left[\max_i P_{\mu,i} \leq P_{s,\max} \right] \mathbb{E}[C_{s,\text{low}}^{(1)}] + \Pr \left[\max_i P_{\mu,i} > P_{s,\max} \right] \mathbb{E}[C_{s,\text{low}}^{(2)}] \quad (23)$$

where $\mathbb{E}[C_{s,\text{low}}^{(1)}]$ and $\mathbb{E}[C_{s,\text{low}}^{(2)}]$ are the average achievable capacities when there is no element in \mathcal{S} and when there exists at least one element in \mathcal{S} and given, respectively, by

$$\begin{aligned} \mathbb{E}[C_{s,\text{low}}^{(1)}] &= \frac{\lambda}{\mu_{\min}} (1 - p_d) \\ &\times \mathbb{E} \left[\log_2 \left(1 + \frac{\max_i \frac{K \beta_{s,i}}{\alpha_{s,i}}}{1 + \beta_p P_p} \right) \middle| \max_i \frac{K}{\alpha_{s,i}} \leq P_{s,\max} \right] \\ &+ \left(1 - \frac{\lambda}{\mu_{\min}} \right) (1 - p_f) \\ &\times \mathbb{E} \left[\log_2 \left(1 + \max_i \frac{K \beta_{s,i}}{\alpha_{s,i}} \right) \middle| \max_i \frac{K}{\alpha_{s,i}} \leq P_{s,\max} \right], \end{aligned} \quad (24)$$

$$\begin{aligned} \mathbb{E}[C_{s,\text{low}}^{(2)}] &= \frac{\lambda}{\mathbb{E} \left[\mu(\alpha_{s,i^\dagger}, P_{s,\max}) \middle| \max_i P_{\mu,i} > P_{s,\max} \right]} (1 - p_d) \\ &\times \mathbb{E} \left[\log_2 \left(1 + \frac{\max_{i \in \mathcal{S}} \beta_{s,i} P_{s,\max}}{1 + \beta_p P_p} \right) \right] \\ &+ \left(1 - \frac{\lambda}{\mathbb{E} \left[\mu(\alpha_{s,i^\dagger}, X P_{s,\max}) \middle| \max_i P_{\mu,i} > P_{s,\max} \right]} \right) \\ &\times (1 - p_f) \mathbb{E} \left[\log_2 \left(1 + \max_{i \in \mathcal{S}} \beta_{s,i} P_{s,\max} \right) \right]. \end{aligned} \quad (25)$$

As N grows, the distribution of maximum random variables in (24) and (25) converges to the Gumbel distribution or Fréchet distribution [17]. Using these approximated distributions, we can asymptotically approximate the expectations as shown in (14).

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